MODELLING THE COMPETITION FROM VIRUNGA MOUNTAIN GORILLA AND GOLDEN MONKEY IN THE VOLCANOE NATIONAL PARK.

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Abstract: In ecological problems, different species interact with each other. Systems of differential equations are used to model these interactions. In this paper, we present a reaction diffusion system which permits to model the competition from two species: Virunga Mountain Gorilla and Golden Monkey with constant diffuse coefficients. The mountain gorilla is one of the wildlife species found in two isolated populations, one about 480 individuals among the volcanoes of the Virunga Massif at the border of Democratic Republic of Congo (DRC), Rwanda and Uganda, the other about 300 individuals in Bwindi Impenetrable National Park in southwest Uganda on the border with DRC. The golden monkey about 500 individuals is also found in the Virunga National Park. Gorillas are in competition with Golden monkey for food. The model use partial differential equations (PDEs). The equilibrium Solutions are determined and their stabilities are examined using Turing procedure. Using the community or stability matrix we prove that all nontrivial steady states result in species elimination. That is, for a long time, one of the species will disappear if nothing is done to protect them. Finally, a numerical simulation is computed using data on Gorilla and Golden Monkey dynamics. Considering the actual growth rate of Mountain Gorilla and Golden monkey respectively and the initial populations, the results show that the competition from the two species for food and living space in the same protected area will result in the extinction of Gorilla

1. INTRODUCTION

1.1. Mountain Gorilla and Golden Monkey

The mountain gorilla is found in two isolated populations, one about 480 individuals (ICCN/UWA/RDB , 2011) among the volcanoes of the Virunga

Massif at the border of DRC, Rwanda and Uganda, the other about 300 individuals in Bwindi Impenetrable National Park in southwest Uganda on the border with DRC (McNeilage, A.et al., 2001). The virungas region includes Mgahinga Gorilla national Park (Uganda), national park of Virunga (Rwanda) and park National des Virunga (Democratic Republic of Congo)
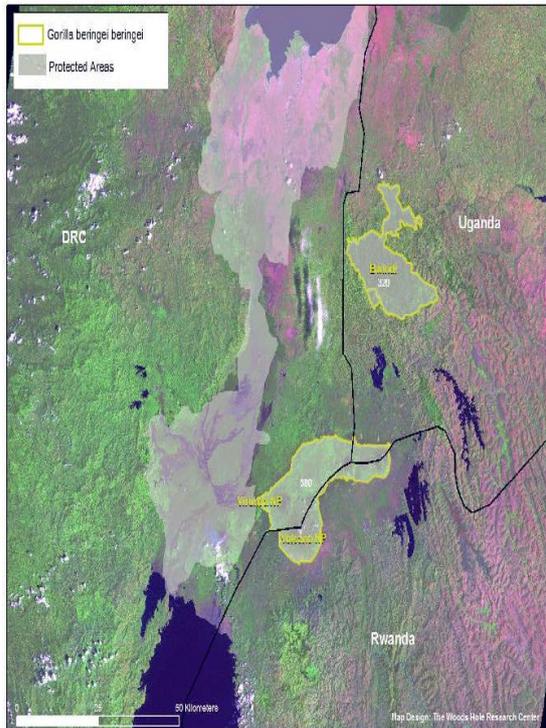

Figure 1. Mountain Gorilla (Gorilla beringei beringei) Location

Figure 2: Gorilla in the Virunga Volcanoe s' Massif

Interest in Virunga Mountain gorilla population has significantly increased since Dian Fossey started their study in 1967: many censuses have been conducted; many research topics have been done. One can mention here the taxonomy, social behaviour, and interaction with the local human population, social and economic impact, disease transmission, and recently some simulation population's dynamics have been developed (Robbins M. (1995, 2004), Weber, A.et al., (1983) . These simulations have been developed using Vortex population Viability Analysis model (Lacy, R.C and Miller P.S., 2005).

Among these studies no competition from other species has been done. Here we consider the competition from

Virunga Mountain Gorilla and Golden monkey.

Figure 3. **Golden Monkey in the Virunga Volcanoe Massif**

The golden monkey (*Cercopithecus mitis khanti*) is a subspecies of blue monkey found only in the bamboo forests of the Virunga Volcanoes Massif in Central Africa (Butynski, T. M., 2008).   Two groups of golden monkeys are habituated for tourism in Volcanoes National Park in Rwanda and one group is habituated in Mgahinga Gorilla National Park . Golden monkeys live in families. The number of members per group varies between 30 to 75 individuals (Groves C. 2005). This includes one dominant male, females, juveniles and babies. The dominant males and adult females are responsible to protect the territory and resources found in, mostly females defend the food. A dominant male leads the group from one to ten years, coordinates all activities in the group, decides where to go where to stand and what to do. Two groups are regularly identified in Volcanoes national park such as: Musonga group located on Karisimbi's slopes with 75 individuals and Kabatwa group located in Sabyinyo slopes with 70 individuals. This is the mostly visited and much habituated group. All golden monkey feed on fresh bamboo shoots. This is the same food for Gorillas. Since they live in the same protected area they are in a competition for food and living space.

1.2. **Objective of the research**

The main objective of this research is the application of differential equation to solve a problem of real life; in this case the competition from two species namely Virunga mountain Gorilla and Golden monkey

The specific objective of this research is to derive a mathematical model that permits to analyze the competition from these two species , to find the equilibrium points and to analyze the behaviour of the dynamics of the two species around the stationary points and to find the impact of this

competition in terms of population dynamics in long term

1.3 **Reaction diffusion system**

In ecological problems, different species interact with each other, and in chemical reactions, different substances react and produce new substances. Systems of differential equations are used to model these events. Similar to scalar reaction diffusion equations, reaction diffusion systems can be derived to model the spatial-temporal phenomena. Suppose that $P(t,x)$ and $Q(t,x)$ are population density functions of two species or concentration of two chemicals, then the reaction diffusion system can be written as (Jumping, 2005)

$$\frac{\partial P}{\partial t} = d_P \frac{\partial^2 P}{\partial x^2} + F(P,Q)$$

$$\frac{\partial Q}{\partial t} = d_Q \frac{\partial^2 Q}{\partial x^2} + G(P,Q) \quad (1)$$

where d_P and d_Q are the diffusion constants of P and Q respectively, and $F(P,Q)$ and $G(P,Q)$ are the growth and interacting functions respectively.

For simplicity, we only consider one-dimensional spatial domains. The domain can either be the whole space

$(-\infty, \infty)$ when considering the spread or invasion of population in a new territory, or a bounded interval say (0, L). Appropriate boundary conditions need to be added to the system to comprise a well posed problem. Typically we consider the Dirichlet boundary condition (hostile exterior):

$$P(t,0) = P(t,L) = 0,$$

$$Q(t,0) = Q(t,L) = 0, \quad (2)$$

or Neumann boundary condition (no-flux):

$$P_x(t,0) = P_x(t,L) = 0,$$

$$Q_x(t,0) = Q_x(t,L) = 0. \quad (3)$$

1.4. Model standard form

The model (1) can be written after a non dimensionalization procedure to a standard form:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \lambda f(u,v)$$

$$\frac{\partial v}{\partial t} = d \frac{\partial^2 v}{\partial x^2} + \lambda g(u,v) \quad (4)$$

or in matrix form as follows

$$\frac{\partial U}{\partial t} = A \Delta U + F(U) \quad (5)$$

where $U = [u,v]^T$ is a column vector representing the densities of the species,

$A = \begin{pmatrix} 1 & 0 \\ 0 & d \end{pmatrix}$ represents the diffusion

matrix and $F = \lambda \begin{pmatrix} f \\ g \end{pmatrix}$ the reaction term

The model (4) is also a particular case of the general model defined by (Nikitin A.G, 2007)

$$\frac{\partial u}{\partial t} - A \, \Delta u = f \qquad (6)$$

where A is the matrix whose elements are

$a_{11}, a_{12}, a_{21}, a_{22}$ and $f = \left(f^1, f^2 \right)^T$

Mathematical models based on equations (6) are widely used in mathematical physics and mathematical biology. Some of these models have been discussed by Nikitin A.G et al. (2006, 2007) and the entire collection of such models is presented by Murray J.D. (1991)

1.5. Turing instability and pattern formation

We consider the initial/boundary value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + \lambda f\left(u,v\right), t > 0, x \in \left(0,1\right) \\[2mm] \dfrac{\partial v}{\partial t} = d\,\dfrac{\partial^2 v}{\partial x^2} + \lambda g\left(u,v\right), t > 0, x \in \left(0,1\right) \\[2mm] u_x\left(t,0\right) = u_x\left(t,1\right) = v_x\left(t,0\right) = v_x\left(t,1\right) = 0 \\[2mm] u\left(0,x\right) = a(x), v\left(0,x\right) = b(x) \end{cases}$$

$$(7)$$

We suppose that $\left(u_0, v_0\right)$ is a constant equilibrium solution, i.e.

$$f\left(u_0,v_0\right) = 0 \text{ and } g\left(u_0,v_0\right) = 0 \quad (8)$$

In fact, the constant solution $\left(u_0, v_0\right)$ is also an equilibrium solution of the system of ordinary differential equations:

$$\begin{cases} u' = \lambda f(u,v), t > 0 \\ v' = \lambda g(u,v), t > 0 \\ u(0) = a, v(0) = b \end{cases} \quad (9)$$

In 1952, Alain Turing published a paper "The chemical basis of morphogenesis" which is now regarded as the foundation of basis chemical theory or reaction diffusion theory of morphogenesis. Turing suggested that, under conditions, chemicals can react and diffuse in such a way as to produce non-constant equilibrium solutions, which represent spatial patterns of chemicals or morphogen concentration. Turing's idea is a simple but profound one. He said that if the absence of the diffusion (considering ODE (9)), u and v tend to a linearly stable uniform steady state, then, under certain conditions, the uniform steady state can become unstable, and spatial inhomogeneous patterns can evolve through bifurcation. In other words, a constant equilibrium can be asymptotically

stable with respect to (9), but it is unstable with respect to (7). Therefore this constant equilibrium solution becomes unstable because of the diffusion, which is called diffusion driven instability.

Theorem   If $(u*, v*)$ is stable with respect to (7), then the eigenvalues of the Jacobian $J = \begin{pmatrix} f_u & f_v \\ g_u & g_v \end{pmatrix}$ (10)

at $(u*, v*)$ must all have negative real part, which is equivalent to

$$Trace(J) = f_u + g_v < 0,,$$

$$Determinant (J) = f_u g_v - f_v g_u > 0.$$

2. Modelling the competition form Virunga Mountain Gorilla and Golden Monkey

2.1 The model

Let $P(s, y)$ be the density of the Virunga Mountain Gorilla and $Q(s, y)$ be the density of the Golden Monkey in the same protected area at time s and position y . Let d_P and d_Q be the respective diffusion coefficient (supposed to here constant). Let $a, c > 0$ be the maximum growth rates per capita of the species. Suppose that in the absence of the other specie(s), the growth rate of the population is slow down due to the limited resource and then the population may saturate to a maximum level. Let $M, N > 0$ be the carrying capacities of the respective species. It is natural to use a density dependent growth rate per capita and here we use the logistic growth rate together with diffusion of the species. Supposing that each member of the specie interact with each member of the other species, the following is the model flow
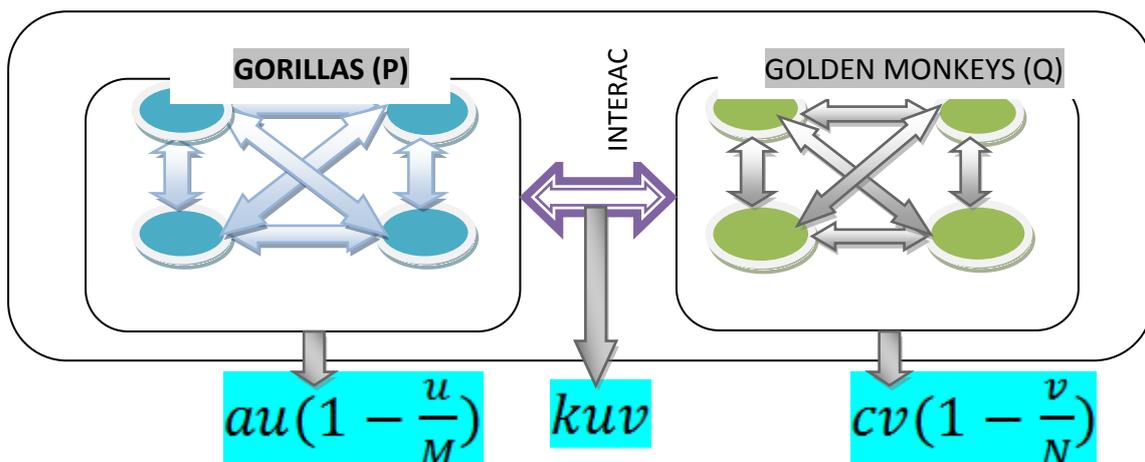

Figure4. **Model Flow Chart**

Then, the growth reaction functions $F(P,Q)$ and $G(P,Q)$ in (1) can be defined as follows

$$F(P,Q) = aP\left(1-\frac{P}{M}\right) - bPQ \quad \text{and}$$

$$G(P,Q) = cQ\left(1-\frac{Q}{N}\right) - dPQ$$

(11)

Since the population is closer to immigration and emigration (no-flux), we add the Neumann boundary conditions. With the initial conditions, the competitive model reduces

$$\frac{\partial P}{\partial s} = d_P \frac{\partial^2 P}{\partial y^2} + aP\left(1-\frac{P}{M}\right) - bPQ$$

$$\frac{\partial Q}{\partial s} = d_Q \frac{\partial^2 Q}{\partial y^2} + cQ\left(1-\frac{Q}{N}\right) - dPQ$$

$$P_y(s,0) = P_y(s,L) = Q_y(s,0) = Q_y(s,L) = 0$$

$$P(0,y) = P_0(y); Q(0,y) = Q_0(y)$$

(12)

2.2. Nondimensionalization procedure

Let consider the Initial boundary value problem (12). The method of separation of variables fails here to find an explicit expression of the solution to the system and that is not surprised due to the nonlinear nature of the system.

In both qualitative and numerical methods, the dependence of solutions of the

parameters plays an important role and there are always more difficulties when there are more parameters. Hence, we usually need to reduce the number of parameters by converting the system into non-dimensionized version. After a non dimensionalization procedure, the system (12) reduces to

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{L^2}{d_P}\left[au(1-u) - bN(uv)\right] \\[2mm] \dfrac{\partial u}{\partial t} = \dfrac{d_Q}{d_P}\dfrac{\partial^2 v}{\partial x^2} + \dfrac{L^2}{d_P}\left[cv(1-v) - dM(uv)\right] \\[2mm] u_x(t,0) = u_x(t,1) = v_x(t,0) = v_x(t,1) = 0 \\[2mm] u(0,x) = u_0(x); v(0,x) = v_0(x) \\[2mm] \qquad s > 0, \ y \in (0,L) \quad (13) \end{cases}$$

that implies the following standard form

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + \lambda f(u,v), \quad t > 0 \ \ x \in (0,1) \\[2mm] \dfrac{\partial v}{\partial t} = d\dfrac{\partial^2 v}{\partial x^2} + \lambda g(u,v), t > 0 \ \ x \in (0,1) \\[2mm] u_x(t,0) = u_x(t,1) = v_x(t,0) = v_x(t,1) = 0 \\[2mm] u(0,x) = a(x), v(0,x) = b(x) \\[2mm] \qquad\qquad\qquad\qquad\qquad (14) \end{cases}$$

where

$$d = d_Q / d_P \; ; \lambda = \frac{L^2}{d_P}$$

$$f(u,v) = (a - au - bNv)$$
$$g(u,v) = v(c - cv - dMu) \quad (15)$$

In matrix form, the system (14) takes the form

$$\frac{\partial w}{\partial t} = D \Delta w + F(w) \quad (16)$$

where $w = [u,v]^T$ is a column vector representing the densities of the species, $D = \begin{pmatrix} 1 & 0 \\ 0 & d \end{pmatrix}$ represents the diffusion matrix and $F = \lambda \begin{pmatrix} f \\ g \end{pmatrix}$ the growth-reaction term

The model (16) is also a particular case of the general model defined by (Nikitin A.G, 2007)

$$\frac{\partial u}{\partial t} - A \Delta u = f(u) \quad (17)$$

where A is the matrix whose elements are $a_{11}, a_{12}, a_{21}, a_{22}$ and $f = \left(f^1, f^2 \right)^T$. For the solvability (existence and uniqueness) of the problem (17) see Evans L., (1999)

2.3. Steady States and stability

The steady states, and phase plane singularities, u^*, v^* are solutions of

$$\frac{du}{dt} = 0; \frac{dv}{dt} = 0 \text{ or from (9) } f(u,v) = 0$$

and $g(u,v) = 0. \quad (18)$

in this case we obtain the system

$$f(u,v) = u(a - au - bNv) = 0$$
$$g(u,v) = v(c - cv - dMu) = 0 \quad (19)$$

Let $a_{12} = \frac{bN}{a}$ and $a_{21} = \frac{dM}{c}$ e be the contributor parameters (20)

Then

$$f(u,v) = 0 \Leftrightarrow u(1 - u - a_{12}v) = 0$$
$$g(u,v) = 0 \Leftrightarrow v(1 - v - a_{21}u) = 0 \quad (21)$$

The equilibrium solutions are

$$u^* = 0, v^* = 0; \quad u^* = 0, v^* = 1; \quad u^* = 1, v^* = 0;$$
$$and \; u^* = \frac{1 - a_{12}}{1 - a_{12}a_{21}}, v^* = \frac{1 - a_{21}}{1 - a_{12}a_{21}} \quad (22)$$

The last is only of relevance if $u^* \geq 0$ and $v^* \geq 0$ are finite, in which case $a_{12}a_{21} \neq 1$. The four possibilities are seen immediately on drawing the null clines $f = 0$ and $g = 0$ in the u, v phase plane as shown in the following figure. The crucial part of the null clines are, from (22), the straight lines $1 - u - a_{12}v = 0$ and $1 - v - a_{21}u = 0$.
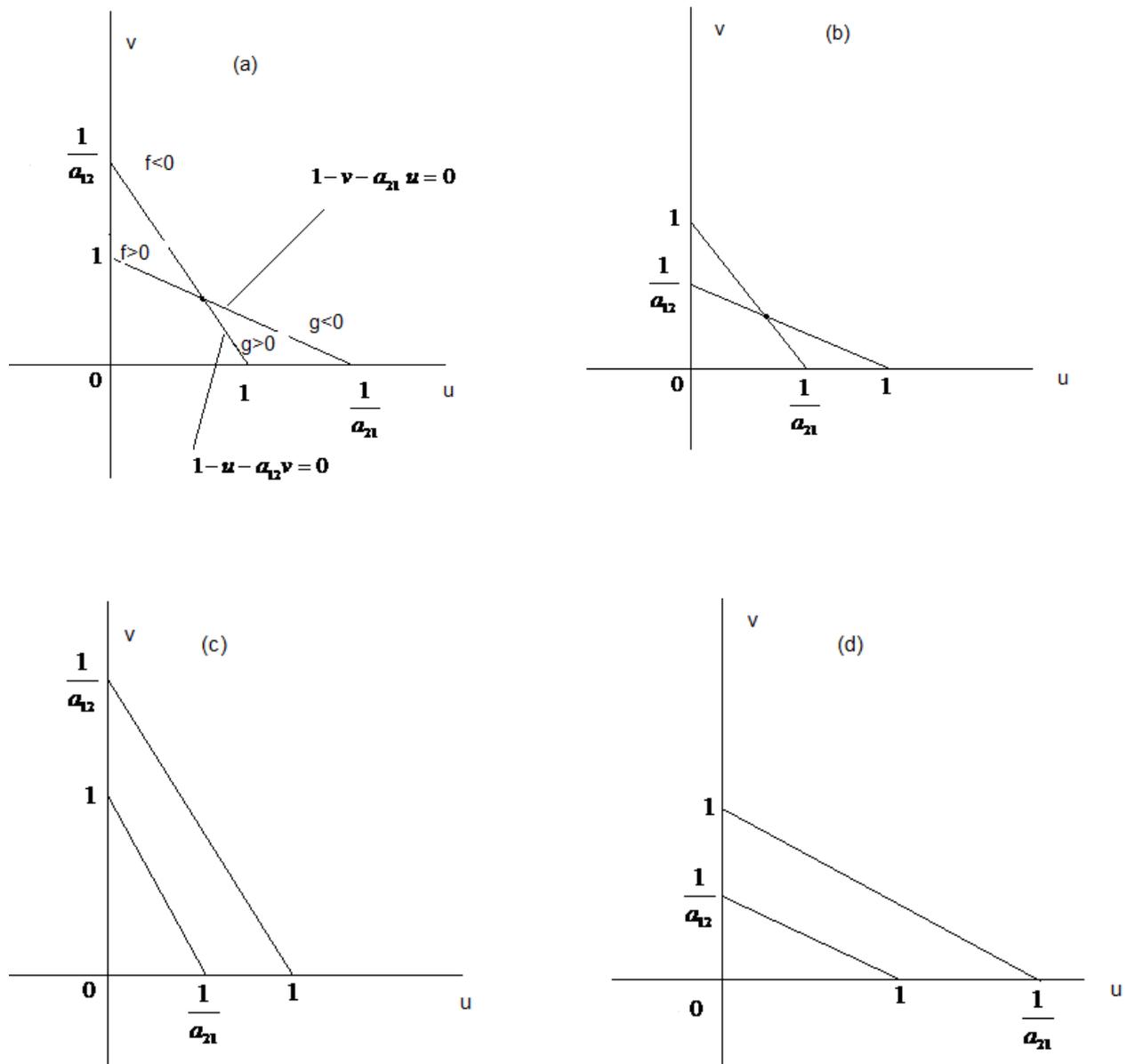
Figure 5: The null clines for the competition model (22). $f = 0$ is $u = 0$ and $1 - u - a_{12}v = 0$ with $g = 0$ being $v = 0$ and $1 - v - a_{21}u = 0$. The intersection of the two solid lines gives the positive steady state if it exists as in (a) and (b).

The stability of the steady states is determined by the community matrix

$$A = \begin{pmatrix} \dfrac{\partial f}{\partial u} & \dfrac{\partial f}{\partial v} \\ \dfrac{\partial g}{\partial u} & \dfrac{\partial g}{\partial v} \end{pmatrix}_{(u^*,v^*)} = \begin{pmatrix} 1-2u-a_{12}v & -a_{12}u \\ -a_{21}v & 1-2v-a_{21}u \end{pmatrix}_{(u^*,v^*)}$$

(23)

The first steady state, that is $(0,0)$ is unstable since the eigenvalues μ of its community matrix $\mu_1 = 1; \mu_2 = 1$ are positive (24)

For the second steady state, namely $(0,1)$, the eigenvalues of the community matrix are

$\mu_1 = 1 - a_{12}; \quad \mu_2 = -1$. And so $u^* = 0; v^* = 1$ is stable if $a_{12} > 1$ and unstable if $a_{12} < 1$. (25)

Similarly, for the third steady state, $u^* = 1; v^* = 0$ the eigenvalues are $\mu_1 = -1; \mu_2 = 1 - a_{21}$ and so it is stable if $a_{21} > 1$ and unstable if $a_{21} < 1$

(i) $a_{12} < 1; \quad a_{21} < 1$

(ii) $a_{12} > 1; \quad a_{21} > 1$

Finally, for the last steady state $\left(\dfrac{1-a_{12}}{1-a_{12}a_{21}}, \dfrac{1-a_{21}}{1-a_{12}a_{21}} \right)$, the eigenvalues are the roots of the following quadratic equation in μ

$$P(\mu) = \mu^2 - \frac{a_{12}+a_{21}-2}{1-a_{12}a_{21}}\mu + \frac{(a_{12}-1)(a_{21}-1)(1+a_{12}a_{21})}{[1-a_{12}a_{21}]^2} = 0$$

(26)

$$\mu_{1,2} = \frac{1}{2}\left\{ \frac{(a_{12}+a_{21}-2) \pm \sqrt{(a_{12}+a_{21}-2)^2 - 4(a_{12}-1)(a_{21}-1)(1+a_{12}a_{21})}}{(1-a_{12}a_{21})} \right\}$$

(27)

The sign of μ, or $\mathrm{Re}(\mu)$ if complex, and hence the stability of the steady state, depends on the size of a_{12} and a_{21}. The various cases are:

(iii) $a_{12} < 1; \quad a_{21} > 1$

(iv) $a_{12} > 1; \quad a_{21} < 1$

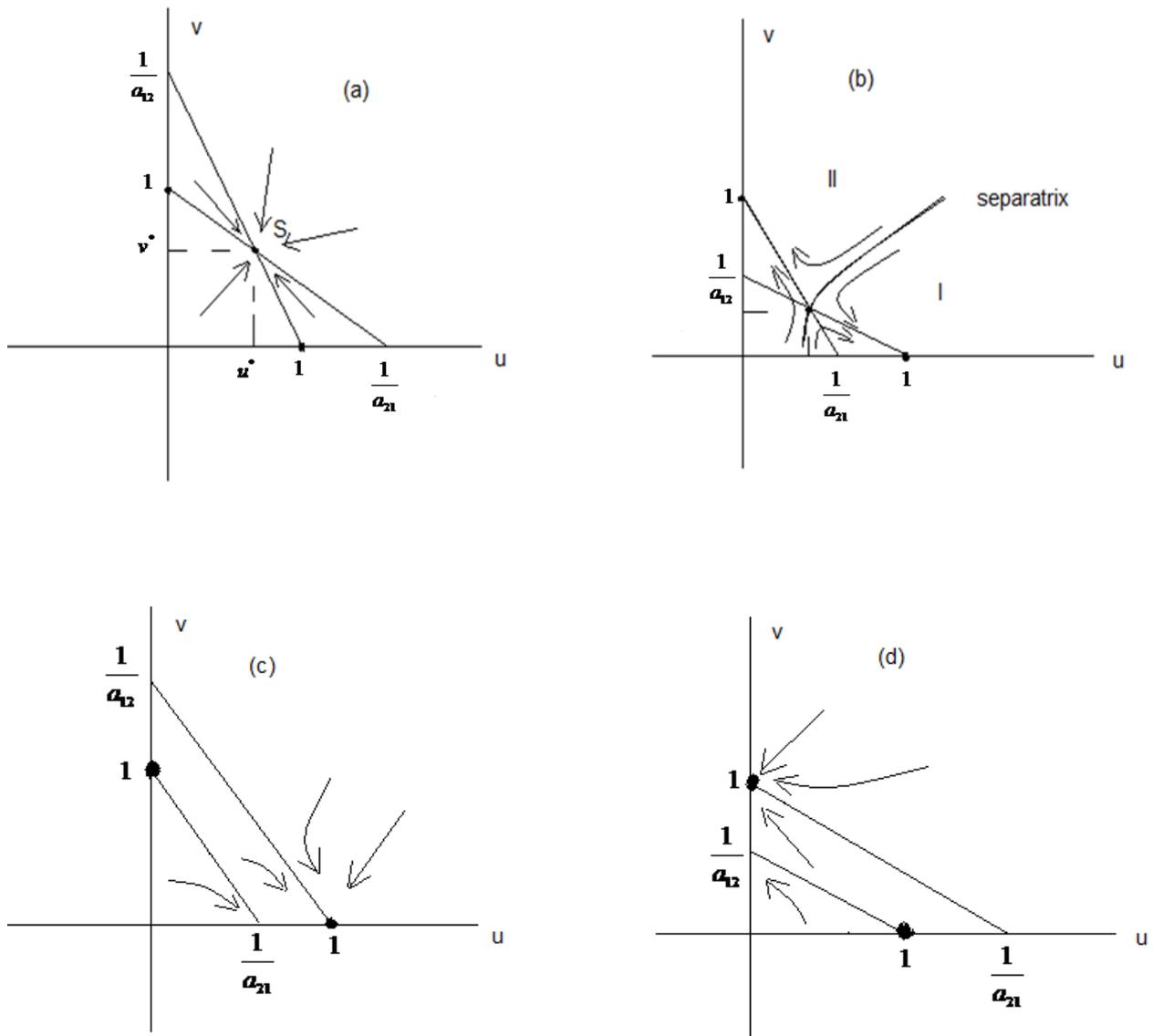The figure below relate these cases (i) to (iv) respectively

Figure 6: Schematic phase trajectories near the steady states for the dynamic behaviour of competing populations satisfying the model (20) for the various cases. (a) $a_{12} < 1; \quad a_{21} < 1$. Only the positive steady state S is stable and all trajectories tend to it. (b) $a_{12} > 1; \quad a_{21} > 1$.

Here $(1,0)$ and $(0,1)$ are steady state each of which has a domain of attraction separated by separatrix that passes trough $(u*, v*)$ which is in this case one of the saddle point trajectories in fact. For the case (c) where $a_{12} < 1; \quad a_{21} > 1$, only one stable steady state exists. $u* = 1; \quad v* = 0$ with the whole positive quadrant its domain of attraction. For the case

(d) $a_{12} > 1;$ $a_{21} < 1$, the only stable steady state is $u^* = 0;$ $v^* = 1$ with the positive quadrant as its domain of attraction. Case (a) illustrate a mutualism where both species can cohabit. Cases (b) to (d) illustrate the competitive exclusion principle whereby 2 species competing for the same limited resource cannot in general coexist.

3. Results Interpretations

Consider some of ecological implications of these results. In case (i), where $a_{12} < 1$ and $a_{21} < 1$, there is a stable steady state where both species can exist as in figure 1 (a). In terms of the original parameters from (20) this correspond to

$$\frac{bN}{a} < 1 \Rightarrow N < \frac{a}{b} \qquad \text{and}$$

$$\frac{dM}{c} < 1 \Rightarrow M < \frac{c}{d} \qquad (28)$$

Thus, if M and N satisfy (28) where the interspecific competition, as measured by b and d is not too strong, a and c representing the respective growth rate, these conditions say that the two species simply adjust to a lower population size than if there were no competition. In other words, the competition is not aggressive.

In case (ii), where $a_{12} > 1$ and $a_{21} > 1$, the analysis says that the competition is such that all three nontrivial steady states can exist, but $(1, 0)$ and $(0, 1)$ are stable, as in figure 1(b). It can be a delicate matter which ultimately wins out. It depends crucially on the starting advantage each species has. If the initial conditions lie in domain I then eventually species 2 will die out, $v \rightarrow o$ and $u \rightarrow 1$;   that is $P \rightarrow M$ the carrying capacity of the environment for P. Thus competition here has eliminated   the Q-species. On the other hand, if Q has an initial size advantage so that u and v start in region II then $u \rightarrow o$ and $v \rightarrow 1$ in which case the P-species becomes extinct and $Q \rightarrow N$, its environmental carrying capacity. We expect extinction of one species even if the initial populations are close to the separatrix and in fact if they lie on it, since the ever present random fluctuations will inevitably cause one of u and v to tend to zero.
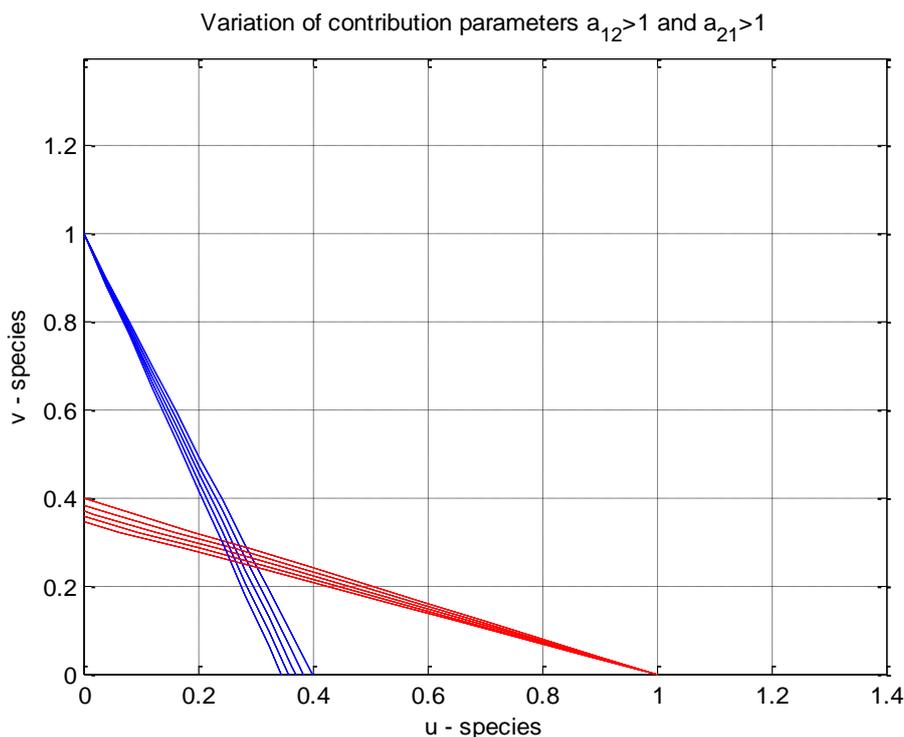
Figure7. Example of variation of the parameters a_{12} and a_{21}

Cases (iii), as in figure 2 (c), the stronger dimensionless interspecific competition of the u-species dominates and the other species, v, dies out. In case (iv) it is the other way round and species u becomes extinct. Although all cases do not result in species elimination, those in (iii) and (iv) always do and in (ii) it is inevitable due to natural fluctuations in the population levels. This is work led to the principle of competitive exclusion which states : when two species compete for the same limited resources one of the species usually becomes extinct.

Note that the conditions for this to hold depend on the dimensionless parameter groupings a_{12} and a_{21} which depend on the interspecific competition, the carrying capacities and the growth rates. The situation in which $a_{12} = 1 = a_{21}$ is special and, with the usual stochastic variability in nature, is unlikely in the real world to hold exactly. In this case the competitive

exclusion of one or the other of the species also occurs.

Note also that all those stable steady states are unstable with respect to the problem (13) due to the diffusion term. For these steady states the method of separation of variables can be applied to find the analytical solution of the Initial / boundary value problem (13)

4. Numerical Applications

4.1. Carrying Capacities of Mountain Gorilla and Golden Monkeys.

The carrying capacities, M, N define the upper limit for the population size. Previous estimates of carrying capacity for the Virunga park region were derivered from Schaller's initial observations (Weber and Vedder, 1986) and were extended by McNeilage (1995). All baseline Virunga models included a carrying capacity of $M = 650$ individuals. No such studies have been undertaken for the Golden Monkey in the Virunga area, hence for this habitat, we take a carrying capacity equal to $N = 1000$ individuals.

4.2. Growth rates

The last census (2010) permits us to consider the maximum annual growth rate for the Virunga Mountain Gorilla equal to $a = 3.75\%$ and the annual growth rate for the golden monkey to be estimated to $b = 5\%$

Let take for the simplicity $b = d = 1$,

4.3. Steady states and stability

The functions $f(u,v)$ and $g(u,v)$ in (20) become

$$f(u,v) = u(0.0375 - 0.0375u - 1000v)$$
$$g(u,v) = v(0.05 - 0.05v - 650u)$$

$$(29)$$

$$f(u,v) = 0 \Leftrightarrow u\left(1 - u - \frac{1000}{0.0375}v\right) = 0$$

$$g(u,v) = 0 \Leftrightarrow v\left(1 - v - \frac{650}{0.05}u\right) = 0$$

$$(30)$$

According to the above discussion, $a_{12} = \dfrac{1000}{0.0375} > 1$ and $a_{21} = \dfrac{650}{0.05} > 1$, we find the case (ii).All three nontrivial steady states exist but only the stable steady state are $(1,0)$ and $(0,1)$ and depending on the starting populations, growing rates and the level of competition, one of the two species will go to extinction and in this case, Gorilla will disappear

5. Concluding remarks

Partial differential equations have been used to model the competition from two species, namely Virunga mountain Gorilla and the golden Monkeys. By using a nondimensionalization procedure, we have found the standard form of the problem (12). We have proved the solvability (existence and uniqueness of weak solution) using the nonvariational techniques. Finally we have found and analysed the stability of all possible equilibrium solutions using the community matrix. All nontrivial steady states result in species elimination. The conditions for this to hold depend on the dimensionless parameter grouping a_{12} and a_{21} defined by (20). When $a_{12} < 1$ and $a_{21} < 1$, only the positive steady state S is stable and all trajectories tend to it. These conditions say that the two species simply adjust to a lower population size than if there were no competition. In other words, the competition is not aggressive.

When $a_{12} > 1$ and $a_{21} > 1$, $(1,0)$ and $(0,1)$ are steady state each of which has a domain of attraction separated by separatrix which passes trough

(u^*, v^*) which is in this case one of the saddle point trajectories in fact. We expect extinction of one species even if the initial populations are close to the separatrix and in fact if they lie on it, since the ever present random fluctuations will inevitably cause one of u and v to tend to zero. If $a_{12} < 1$ and $a_{21} > 1$, only one stable steady state exists $u^* = 1;\quad v^* = 0$ with the whole positive quadrant its domain of attraction. The stronger dimensionless interspecific competition of the u -species dominates and the other species, v, dies out. At the end, when $a_{12} > 1$ and $a_{21} < 1$, the only stable steady state is $u^* = 0;\quad v^* = 1$ with the positive quadrant as its domain of attraction, it is the other way round and species u become s extinct. We conclude that: when two species compete for the same limited resources and if the competition is unbalanced one of the species usually becomes extinct.

To save the species in competition, I recommend separating them where it is possible. If not we must be sure that one of the specie will disappear with time. For example to move the Golden in NYUNGWE National Park (south of Rwanda)

Acknowledgement

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APPENDIX

Table : List of variables, dimensions, parameters and their dimensions

Variables	Dimension	Parameter	Dimension
s	τ	D_P	$\tau^{-1}\theta^2$
y	θ	D_Q	$\tau^{-1}\theta^2$
P	μ	a	μ^2
Q	γ	b	μ
		c	γ^2
		d	γ
		L	θ
		M	μ
		N	γ

To reduce the number of parameters we introduce dimensionless variables

$$t = \frac{d_P}{L^2}s, \qquad s > 0 \Leftrightarrow t > 0$$

$$x = \frac{y}{L} \qquad y \in (0,L) \Leftrightarrow x \in (0,1)$$

$$u = \frac{P}{M}; v = \frac{Q}{N}$$

(1)

Using the new variables, we have

$$\frac{\partial P}{\partial s} = \frac{\partial P}{\partial u} \cdot \frac{\partial u}{\partial t} \cdot \frac{\partial t}{\partial s} = M \frac{d_P}{L^2} \frac{\partial u}{\partial t}$$

(2)

$$\frac{\partial Q}{\partial s} = \frac{\partial Q}{\partial v} \frac{\partial v}{\partial t} \frac{\partial t}{\partial s} = N \frac{d_P}{L^2} \frac{\partial v}{\partial t}$$

(3)

$$\frac{\partial^2 P}{\partial y^2} = \frac{\partial P}{\partial u} \frac{\partial^2 u}{\partial x^2} \left(\frac{\partial x}{\partial y}\right)^2 = M \frac{1}{L^2} \frac{\partial^2 u}{\partial x^2}$$

(4)

$$\frac{\partial^2 Q}{\partial y^2} = \frac{\partial Q}{\partial v} \frac{\partial^2 v}{\partial x^2} \left(\frac{\partial x}{\partial y}\right)^2 = N \frac{1}{L^2} \frac{\partial^2 v}{\partial x^2}$$

(5)